**ECE 7650 (Advance Matrix Algorithm)**

**REPORT**

*On*

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*By*

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**PROBLEM 1**

**Question 1(i)**

This question was implemented in file named “**Q1i.m**”. Random matrices of chosen dimension were used and the following results were obtained:

**Case 1:** For randomly chosen ‘**m**’

|  |  |  |
| --- | --- | --- |
| Matrix Dimension | Size of Krylov Subspace (m) | Rank of V |
| 10x10 | 5 | 6 |
| 20x20 | 12 | 13 |
| 50x50 | 35 | 12 |
| 100x100 | 98 | 8 |
| 200x200 | 131 | 7 |
| 300x300 | 211 | 7 |
| 400x400 | 313 | 1 |

**Case 2:** For fixed ‘**m**’ for different dimensions of input matrix **A**

|  |  |  |
| --- | --- | --- |
| Matrix Dimension | Size of Krylov Subspace (m) | |
| **Rank(V) for m=5** | **Rank(V) for m=10** |
| 10x10 | 6 | 10 |
| 20x20 | 6 | 11 |
| 50x50 | 6 | 11 |
| 100x100 | 6 | 11 |
| 200x200 | 6 | 11 |
| 300x300 | 6 | 11 |
| 400x400 | 6 | 11 |

**Observation:**

As it can be observed from the tables, it is obvious that choosing **V = [r Ar A2r . . . Am-1r]** resulted in linearly dependent bases **V** such that, **V** is rank deficient. While **Case 1** presented above tends to be a generic case and not totally clear, however observing **Case 2** actually led to the conclusion that for a chosen size of Krylov subspace ‘**m**’, provided that **m < dim(A)**, **rank(V) = m+1** and for **m = dim(A)**, **rank(V)** = **m** for linearly independent bases.

**Note:** *The ranks obtained are more than ‘****m****’ since dimension of* ***V*** *is actually* ***(m+1)x1*** *but in a specific case* ***Vm*** *of dimension* ***mx1*** *can be obtained from it by just removing the last column.*

**Question 1(ii)**

Arnoldi method was implemented based on Modified Gram-Schmidt process as a function called ‘**arnoldi.m**’. The driver program for this question is named ‘**Q1ii.m**’. For the sake of clarity, ‘**Raw V**’ used to refer to the bases **V** obtained from **V = [r Ar A2r . . . Am-1r]** while the ones obtained from Arnoldi iteration is termed ‘**Arnoldi V**’.

**Case 1:** For randomly chosen ‘**m**’

|  |  |  |  |
| --- | --- | --- | --- |
| Matrix Dimension | Size of Krylov Subspace (m) | Rank of | |
| **Raw V** | **Arnoldi V** |
| 10x10 | 9 | 10 | 10 |
| 20x20 | 11 | 12 | 12 |
| 50x50 | 41 | 13 | 42 |
| 100x100 | 78 | 11 | 79 |
| 200x200 | 153 | 8 | 154 |
| 300x300 | 240 | 4 | 241 |
| 400x400 | 297 | 2 | 298 |

**Case 2:** For fixed ‘**m**’ for different dimensions of input matrix **A**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Matrix Dimension | Size of Krylov Subspace (m) | | | |
| **Rank(V) for m=5** | | **Rank(V) for m=10** | |
| **Raw V** | **Arnoldi V** | **Raw V** | **Arnoldi V** |
| 10x10 | 6 | 6 | 10 | 10 |
| 20x20 | 6 | 6 | 11 | 11 |
| 50x50 | 6 | 6 | 11 | 11 |
| 100x100 | 6 | 6 | 11 | 11 |
| 200x200 | 6 | 6 | 11 | 11 |
| 300x300 | 6 | 6 | 11 | 11 |
| 400x400 | 6 | 6 | 11 | 11 |

**Observation:**

From the table shown above for **Case 1**, it can be observed that using Arnoldi method, the bases **V** obtained are linearly independent such the rank of **V** are now **m+1** (for the reasons previously presented). If **Vm** is considered, that is **V** of dimension **mx1**, then **rank(V) = m**.

Hence forth, the table in **Case 2** perfectly confirm the assertion made in the “**Observation**” section of the previous question, that is Question 1(i) above for **Case 2**.